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CONNECTION BETWEEN VACUUM SYMMETRY,

THE GOLDSTONE THEOREM, AND NEUTRAL PCAC

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ABSTRACT

The distinction between current and constituent quarks is manifested in the inequivalence of vacuum matrix elements of the corresponding bad operators, $\langle 0|(\bar{q}q)_{con}|0\rangle \neq \langle 0|(\bar{q}q)_{curr}|0\rangle$. Since the former relates to the Nambu-Goldstone spontaneous breakdown of chiral symmetry, $\langle 0|(\bar{q}q)_{con}|0\rangle \neq 0$, there is no compelling reason why $\langle 0|(\bar{q}q)_{curr}|0\rangle$ cannot vanish in the chiral-symmetric limit. A consistent "neutral PCAC" picture, in which $\langle 0|(\bar{q}q)_{curr}|0\rangle$ is proportional to the current quark mass, is shown to provide an excellent phenomenological description of the chiral symmetry-broken world. This includes the flavor independence of the dynamically-generated quark mass. Moreover, the U(1) vacuum Ward identity puzzle has a trivial solution in the neutral PCAC scheme.

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I. Introduction

The Goldstone theorem[1] applied to chiral symmetry states that for conserved axial-vector currents (in the chiral limit), if the associated SU(3) charges Q_5^i, $i = 1 - 8$, break the vacuum symmetry.

$$Q_5^i|0\rangle \neq 0 \quad , \tag{1}$$

then there must exist massless π, K and η_8 pseudoscalar mesons in the theory. The condition (1), however, is not easy to implement in practice, and so one usually invokes a slightly stronger condition for spontaneous breakdown of chiral symmetry[2,3]:

$$(0|\bar{q}q|0) \neq 0 \ , \tag{2a}$$

valid for each SU(3) flavor u, d, s. Since the vacuum is a unitary singlet, it is quite natural to extend (2a) in the chiral limit to

$$(0|\bar{u}u|0) = (0|\bar{d}d|0) = (0|\bar{s}s|0) \neq 0. \tag{2b}$$

The controversy is not whether (2) implies (1), but rather, to which type of quark fields do (2) refer. In this paper we suggest, contrary to the implications of refs. 2 and 3, that (2) refers to <u>constituent</u> rather than <u>current</u> quark fields. This proposition has profound implications for the theory of chiral symmetry <u>breaking</u>:

(i) for the current quark mass scales and ratios

(ii) for the accepted additivity connection between constituent and current quark masses

$$m_i^{\text{constituent}} = m_i^{\text{current}} + m^{\text{dynamic}} \tag{3}$$

and the flavor-independence of the latter dynamically-generated quark mass

(iii) for the U(1) problem, i.e. consistency with the U(1) vacuum Ward identity.[4]

To this end, in Sec. II we review the theoretical and phenomenological distinction between current and constituent quarks, especially for the "bad" operators $\bar{q}q$. Then in Sec. III we show how the Nambu[5] dynamical realization of spontaneous breakdown in the chiral limit unambiguously requires constituent quarks to satisfy the vacuum inequality (2a). We apply renormalization-group ideas[6] and Goldberger-Treiman relations at the quark level to reaffirm this connection. From the viewpoint of the Goldstone theorem, the essential content of Secs. II - III is that in the chiral limit

$$\langle 0|\bar{q}q)_{con}|0\rangle \neq 0 \tag{4a}$$

$$\langle 0|(\bar{q}q)_{curr}|0\rangle = 0 \quad . \tag{4b}$$

the latter result being stictly valid in the free quark model.

Given (4), in Sec. IV we turn on the chiral symmetry breaking current quark mass matrix and investigate the differences between "strong" PCAC[2,3] and "neutral" PCAC,[7,8] the latter requiring that (4b) is proportional to one power of current quark mass which does vanish in the chiral limit. Then $m_\pi^2 = \mathcal{O}(\hat{m}^2_{curr})$ combined with bare structure function integrals sets the current quark mass scale of $\hat{m}_{curr} \sim 60$ MeV and also resolves the U(1) problem in a simple way. Finally in Sec. V we compute all current quark masses in order to demonstrate that $m_{con} - m_{curr}$ is indeed flavor independent as suggested in (3). Renormalization group ideas are then used to fortify this result and also to reaffirm the neutral PCAC current quark mass scales The distinction between $\langle 0|(\bar{q}q)_{con}|0\rangle$ and $\langle 0|(\bar{q}q)_{curr}|0\rangle$ is then summarized in Sec. VI.

II. Current vs. Constituent Quarks

Two quark pictures associated with different SU(6) algebras of operators have been quite successful in hadronic physics. "Current" quark fields have been used to construct U(3) quark currents for i = 0 - 8,

$$V^{i}_{\mu}(x) = \frac{1}{2}\,[\bar{q}(x)\ \lambda^{i}\ \gamma_{\mu}\ \bar{q}(x)]_{curr},\quad A^{i}_{\mu}(x) = \frac{i}{2}[\bar{q}(x)\,\lambda^{i}\ \gamma_{\mu}\gamma_{5}\ q(x)]_{curr}\,, \tag{5}$$

which in turn are used to realize the SU(3) x SU(3) current algebra.[9] These $SU(6)_{W,curr}$ currents are also presumed to be directly involved in weak and electromagnetic processes. Furthermore, via PCAC, these current quark operators control various strong interaction processes involving pions and kaons.

On the other hand the "constituent" quark picture seems appropriate for the classification of the low-lying hadrons. Not only can such states be grouped into different irreducible representations of $SU(6)_{W,strong}$, but the additivity of constituent quark masses and magnetic moments give an amazingly accurate picture of the corresponding masses and moments of the composite hadrons.

It is by now well appreciated that current and constituent quark masses are conceptually and numerically different, but it is important to stress that current and constituent quark <u>fields</u> are also not identical. Phenomenological arguments which lead to the latter conclusion include the observations that if one assumes identity then the Adler-Weisberger relations should be well-saturated by the lowest $\frac{1}{2}^{+}$ and $\frac{3}{2}^{+}$ baryon multiplets and that baryon anomalous magnetic moments should vanish. Both of these conclusions are, of course, in conflict with data. In addition, there are purely theoretical arguments which show that an identity between current and constituent quark fields cannot hold; Melosh[10] has argued this on the basis of a conflict with ordinary rotational invariance, Bell and Hey[11] have shown that this identity excludes the existence of the $\underline{35}(L_z = 0)$ mesons, and de Alwis and Stern[12] have proved that this identity implies either hadron mass degeneracy or incorrect singularity structure of the matrix elements of currents. Put another way, composite hadron states built up from constituent but not current quarks are sharp in angular momentum.

Although current and constituent quarks are distinct, it is perhaps not unreasonable that they are related by a unitary transformation. Such a transformation was constructed by Melosh[10] for degenerate, non-interacting quark fields (it is not trivial even then). This Melosh transformation has in fact enlarged the number of theoretical hadronic decay transitions of the form $\alpha \to \beta + \pi$ which are consistent with data[13]. While attempts[14] to generalize the Melosh transformation to interacting quark theories have not altogether been successful, it will be sufficient for our purposes that the two quark pictures are definitely distinct.

One criterion for a unitary Melosh transformation is that it transforms "good" quark operators such as Q^i and Q_5^i into good operators. However, "bad" quark operators such as $(\bar{q}q)_{curr}$ and $(\bar{q}q)_{con}$ are not expected to be unitarily related. In particular it is believed that the vacuum inequality

$$\langle 0|(\bar{q}q)_{curr}|0\rangle \neq \langle 0|(\bar{q}q)_{con}|0\rangle \tag{6}$$

holds true for interacting quarks since it certainly valid for free quarks.

III. Extended Goldstone Theorem and $\langle 0|(\bar{q}q)_{con}|0\rangle$.

To repeat: the Goldstone theorem[1] states that if the (axial) charge Q_5 associated with the conserved current $\partial \cdot A = 0$ does not annihilate the vacuum, $Q_5|0\rangle \neq 0$, then there must exist a massless Nambu-Goldstone boson (NGB) in the theory. The drawback to this theorem is that it is usually difficult to determine directly if $Q_5|0\rangle \neq 0$.

For the case of chiral symmetry, it is believed sufficient that the chiral non-invariant operator $\bar{q}q$ obeys[2]

$$\langle 0|\bar{q}q|0\rangle \neq 0 \rightarrow Q_5|0\rangle \neq 0 \rightarrow \quad \text{NGB} \quad . \tag{7}$$

Even here, however, a subtlety arises because we have stressed in (6) that the vacuum matrix elements of current and constituent bad quark densities are not equal. Thus to clarify the "extended" Goldstone theorem (7), we must specify whether $\bar{q}q$ refers to current or constituent quark fields.

To this end we consider the Nambu[5] dynamical realization of the Goldstone theorem for the spontaneous breakdown of chiral symmetry. Although it was originally demonstrated for a four fermion theory[5], it has been recently extended to chiral-invariant vector gluon-quark theories [15] with the aid of the axial-vector Ward identity. This demonstration amounts to showing that the equations that dress the quark (DE) and give it its mass $m = m_{dyn}$ (assuming that $m_o = 0$ in the chiral limit) are identical to the Bethe-Salpeter dynamical equations which bind the quark-antiquark in s-wave as $q \rightarrow 0$ into a massless pseudoscalar ($\mathrm{PBE}|_{q \rightarrow 0}$):

$$\mathrm{DE} = \mathrm{PBE}|_{q \rightarrow 0} \quad . \tag{8}$$

Rather than repeat the dynamical proof of (8) here, we exploit the axial

Ward identity to obtain the kinematic structure of the resulting axial vertex function. First define the inverse complete quark propagator as

$$S^{-1}(p) = C(p^2) + D(p^2)\not{p} \ , \tag{9}$$

with *off shell or "running" quark mass determined by*[6]

$$m(p^2) = -C(p^2)/D(p^2) \ . \tag{10}$$

Then the axial Ward identity (in an axial gauge and deleting flavor indices)

$$-iq^\mu \Gamma_{\mu 5}(p;q) = S^{-1}(p + \tfrac{1}{2}q)\gamma_5 + \gamma_5 S^{-1}(p - \tfrac{1}{2}q) \tag{11}$$

can be expanded around $q \to 0$ to determine the axial-vector vertex

$$\Gamma_{\mu 5}(p;q) = iD(p^2)[-\frac{2m(p^2)q_\mu\gamma_5}{q^2} + \gamma_\mu\gamma_5] + \dots \ . \tag{12}$$

Clearly the pseudoscalar wave function is proportional to $q^\mu\Gamma_{\mu 5}$ and the first term in (12) exhibits the massless NGB pole. Furthermore since $D(p^2) \neq 0$ (in fact $D(p^2) \to 1$ as $p^2 \to \infty$), if $m(p^2) \neq 0$ then the NGB exists in the theory with in fact the condition (8) satisfied.

The link between the Goldstone and Nambu versions of spontaneous breakdown thus reduces to the relation between $\langle 0|\bar{q}q|0\rangle \neq 0$ in (7) and $m(p^2) \neq 0$ in (12). Since in fact the complete quark propagator in (9) is the Fourier transform of $\langle 0|T(\bar{\psi}(x), \psi(0))|0\rangle$, a short distance and anomalous-dimension analysis extrapolated to low p^2 suggests that[6]

$$\langle 0|\bar{q}q(M)|0\rangle \propto m^3(M), \tag{13}$$

where the operators are renormalized at a fixed mass scale M, to be clarified shortly.

The final step is to identify $m(p^2)$ in (12) and (13) as the constituent quark mass. In the chiral limit this is certainly the case because $m_{curr} \to 0$ in this limit. Furthermore, if the quarks are taken "on-shell" according to the gauge-invariant prescription ($m \to m(M)$)

$$m_{con} = m(p^2 = m_{con}^2), \tag{14}$$

the structure of the Goldberger-Treiman induced pseudoscalar term in (12), $f_\pi g_{\pi qq} = \hat{m} \equiv \frac{1}{2}(m_u + m_d)$ is similar to the nucleon version with $f_\pi g_{\pi NN} = m_N g_A$. The connection between the two cases is the quark additivity relation $m_N \approx 3\hat{m}$, which by definition refers to "free" constituent quarks in (14). To make this point in a more quantitative fashion, consider for the moment the SU(3)-breaking Goldberger-Treiman relations at the quark level which follow from (14),

$$f_\pi g_{\pi qq} = \hat{m}_{con} \quad , \quad f_K g_{Kqq} = \frac{1}{2}(m_s + \hat{m})_{con} \quad . \tag{15}$$

Since one expects $g_{\pi qq}$ and g_{Kqq} to be weakly dependent upon (non-relativistic) reduced masses along with flavor-independent quark-gluon coupling constants, it is reasonable to assume that $g_{\pi qq} \approx g_{Kqq}$ in (15). Then one obtains[15] for $f_K/f_\pi \approx 1.2$,

$$(m_s/\hat{m})_{con} = 2(f_K/f_\pi) - 1 \approx 1.4 \quad , \tag{16}$$

which is precisely the accepted[16] constituent quark mass ratio with $\hat{m} \approx 330$ MeV and $m_s \sim 550$ MeV.

Returning to the chiral limit, the above discussion allows us to sharpen the extended Goldstone theorem to read

$$\langle 0 \mid (\bar{q}q)_{con} | 0 \rangle \neq 0 \to Q_5 | 0 \rangle \neq 0 \to \text{NGB} \ , \tag{17}$$

where the constituent structure of $\bar{q}q$ in (17) follows from (12) - (16). The Nambu version (12) of the Goldstone theorem requires that the equations which dress the constituent quark in the chiral limit (with $m_{curr} = 0$) and give it its mass $m_{con} \neq 0$, also refer to constituent quark fields with $\langle 0 | (\bar{q}q)_{con} | 0 \rangle \neq 0$.

Put another way, the axial-vector quark current in (5) certainly is built up from current quark fields, while the axial-vector constituent quark vertex (12) exhibits spontaneous breakdown of chiral symmetry by displaying the Nambu-Goldstone induced pseudoscalar pole provided $\langle 0|(\bar{q}q)_{con}|0\rangle$ and therefore $m(p^2)$ are non-vanishing. By way of contrast with (12) and (17), operators built up from current quarks in (5) appear to be insensitive to the free quark limit (whereas the constituent quark vertex (12) is not), since the free quark current algebra and Heisenberg equations of motion have the same structure as in the interacting quark case. Thus it is perhaps reasonable to compute $\langle 0|(\bar{q}q)_{curr}|0\rangle$ in a free quark model in the chiral limit (CL) with $m_{curr} = 0$:

$$\langle 0|(\bar{q}q)_{curr}|0\rangle_{free,CL} \propto \int \frac{d^4p \, \mathrm{Tr}\not{p}}{p^2} = 0 \quad . \tag{18a}$$

Furthermore, we may convert the bare vacuum in (18a) to the physical vacuum via quark-gluon interations in the loop. Since the latter γ-matrix couplings are each accompanied by one quark propagator, (18a) is dressed in any finite order or perturbation theory (PT) according to

$$\langle 0 \mid (\bar{q}q)_{curr}|0\rangle_{PT,CL} \propto \int d^4p \; \mathrm{Tr}(\frac{1}{\not{p}} \not{B} \frac{1}{\not{p} + \not{q}_1} \not{B} \frac{1}{\not{p} + \not{q}_2} \cdots \not{B} \frac{1}{\not{p}}) = 0, \tag{18b}$$

because there are always an odd number of γ-matrices in the trace. The point is that spontaneous breakdown is non-perturbative, but chiral symmetry breaking may very well be perturbative.

IV. Chiral Symmetry Breaking and $\langle 0|(\bar{q}q)_{curr}|0\rangle$.

In the real world the π, K and η_8 do have mass, and so one must combine the spontaneous breakdown of chiral symmetry giving $m_\pi = m_K = m_{\eta_8} = 0$ with chiral symmetry breaking corresponding to $m_o \neq 0$ in the Nambu dynamical approach. In particular, one assumes that the NG pseudoscalars acquire all of their mass from quark mass terms m_u^{curr}, m_d^{curr}, m_s^{curr} which appear in the familiar chiral-breaking SU(3) quark Hamiltonian density

$$H' = (\bar{q}\, M q)_{curr} = m_u^{curr}(\bar{u}u)_{curr} + m_d^{curr}(\bar{d}d)_{curr} + m_s^{curr}(\bar{s}s)_{curr}. \tag{19}$$

In (19) we stress that both the masses and fields are current quark quantities.

The next step is to apply PCAC to $\langle\pi|H'|\pi\rangle$, etc., and invoke the SU(3) x SU(3) current algebra to the current quark fields in (19), leading to[2,3]

$$m_\pi^2 \simeq \frac{1}{f_\pi^2} \langle 0|[Q_5^3,\ [Q_5^3,\ H']]|0\rangle = \frac{\hat{m}_{curr}}{f_\pi^2} \langle 0|(\bar{u}u + \bar{d}d)_{curr}|0\rangle \tag{20}$$

It is at this point where we differ from conventional wisdom, according to which the current quark fields in (20) are not distinguished from the constituent quark fields in the extended Goldstone theorem (17). Indeed, we have seen in (18) that $\langle 0|(\bar{q}q)_{curr}|0\rangle$ in the free quark model is radically different from $\langle 0|(\bar{q}q)_{con}|0\rangle$ in the chiral limit.

We have stressed in Sec. III that the constituent quark fields are linked to the static axial charges in the extended Goldstone theorem. By static we refer to vector and axial charges defined on space-like surfaces

$$Q^i = \int d^3x\, V_o^i(x) \qquad Q_5^i = \int d^3x\, A_o^i(x) \quad . \tag{21}$$

The existence of the NG mode $Q_5^i|0\rangle \neq 0$ requires that physical hadron states have complicated transformation properties under the static charges Q_5^i. However the charges defined on the light plane[17]

$$\hat{Q}^i = \int d^2x_\perp \int dx_+ V_-^i(x) \qquad \hat{Q}_5^i = \int d^2x_\perp \int dx_+ A_-^i\,(x) \qquad (22)$$

do annihilate the vacuum[18] $\hat{Q}^i|0\rangle = \hat{Q}_5^i|0\rangle = 0$. Moreover, the light-plane charges $\hat{Q}^i$ transform single-particle hadron states into single-particle hadron states, a statement not true of the static charges Q^i, which generate particle pairs, etc. Thus, the light-plane charges are the natural choice to study the algebraic structure of the hadrons and the "bad" quark operators $(\bar{q}q)_{curr}$. In departing from the chiral limit as in (19) or (20), the light plane charges still annihilate the vacuum; they are thus the natural link to the chiral breaking operators $(\bar{q}Mq)_{curr}$.

This mismatch between hadron states transforming simply under $\hat{Q}^i$, while PCAC is driven by Q_5^i through the spontaneous breakdown mechanism, is reflected in the difference between the SU(3) transformation properties of constituent and current quark bad operators. That is, $(\bar{q}\lambda_i q)_{con}$ transforms simply as λ_i, while $(\bar{q}\lambda_i q)_{curr}$ cannot transform like λ_i as is usually assumed[2,3]. For example in a vector-gluon quark theory, the "good" two-component quark fields ϕ must be projected from the four component current quark fields q_{curr}, leading to[20]

$$(\bar{q}Mq)_{curr} \sim \phi^\dagger \sigma_\perp \cdot (\nabla_\perp + igB_\perp)\, M_{curr} \nabla_-^{-1}\phi$$
$$+ \phi^\dagger M_{curr}^2 \nabla_-^{-1}\phi \quad . \qquad (23)$$

The analog of the free quark-vacuum matrix elements of (23) is (18), which derives from the vanishing of the spin-flip transition to the bare vacuum. To any finite order in perturbation theory, therefore, we have

$$\langle 0|\phi^\dagger \vec{\sigma}_\perp \cdot g\vec{B}_\perp \nabla_-^{-1}\phi|0\rangle = 0, \quad \langle 0|\phi^\dagger \nabla_-^{-1}\phi|0\rangle \sim \mathcal{O}(1), \qquad (24)$$

since light-plane and canonical perturbation expansions are equivalent. Note that vacuum matrix elements suppress the spin-flip first terms of (23) as in the parton model, with "Z-diagram" suppression.(8,21) This spin-flip suppression also occurs for the soft pion transition from $\langle\pi|\bar{q}q|\pi\rangle$ to $\langle 0|\bar{q}q|0\rangle$ in (20). Thus we have, from (23) and (24),

$$\langle 0|(\bar{q}q)_{curr}|0\rangle \approx M_{curr} \langle 0|\phi^{\dagger}\nabla_{-}^{-1}\phi|0\rangle \qquad (25a)$$

$$\rightarrow 0 \text{ in chiral limit,} \qquad (25b)$$

with (25b) recovering the free current quark limit in (18). We shall refer to the explicit current quark mass factor in (25) as a statement of(8) "neutral" PCAC, which in turn requires <u>two</u> powers of current quark mass in (20)

$$m_{\pi}^{2} = O(\hat{m}_{curr}^{2}), \qquad (26)$$

as opposed to <u>one</u> power of current quark mass in the "strong" PCAC picture.(2,3) That is, for neutral PCAC, $(\bar{q}q)_{curr}$ "remembers" only the current quark mass in (25), while for strong PCAC $(\bar{q}q)_{curr}$ "knows" about spontaneous breakdown, $\langle 0|(\bar{q}q)_{curr}|0\rangle \propto m_{con}$. In the latter approach $\langle 0|(\bar{q}q)_{curr}|0\rangle$ is thus assumed to be identical to $\langle 0|(\bar{q}q)_{con}|0\rangle$ and not vanish in the chiral limit.

As in any scheme of PCAC at the hadron level, neutral PCAC at the quark level must involve an extrapolation scale away from the CAC, chiral limit. Either in (26) or in

$$\partial\cdot A^{\pi} = -i[Q_5^3,\ H'] = -\hat{m}_{curr}(\bar{q}\gamma_5\lambda^3 q)_{curr}\ , \qquad (27)$$

with $\langle 0|(\bar{q}\gamma_5\lambda^3 q)_{curr}|\pi\rangle$ also proportional to one power of $\hat{m}_{curr}$ by analogy(8) with (23) and (25), we expect the extrapolation scale to be the constituent quark mass $\hat{m}_{con} \approx 330$ MeV. Then neutral PCAC corrections in (26) and (27) are of order $(\hat{m}_{curr}^2/\hat{m}_{con}^2)$, so that $\hat{m}_{curr}$ need not be as small for neutral PCAC as for strong PCAC, where $\hat{m}_{curr} \approx 5$ MeV.

In order to extract the actual current-quark mass scale for neutral PCAC, we first observe that(22) $\sigma_{\pi\pi}^{\pi\pi} = m_{\pi}^2$ along with (26) requires the Zweig rule PCAC constraint

$$\langle\pi|(\bar{s}s)_{\rm curr}|\pi\rangle \approx 0 \quad . \tag{28}$$

Now in renormalization-group language,[6] $\hat{m}(p^2)$ in (10) is presumed to recover $\hat{m}_{\rm curr}$ provided p^2 is high enough ~ 10 GeV^2 to expose the "bare" current valence quarks in the pion, but without being too large so as to excite quark pairs in the sea, leading to scaling and Zweig rule violations. Thus we speak of a "bare" quark pion structure function h(x) as required in the pion matrix elements of the dominant second term of (23)

$$m_\pi^2 = 2\hat{m}^2_{\rm curr}\tilde{h} \quad , \qquad \tilde{h} = \int_0^1 dx\; h(x)\,/x \quad . \tag{29}$$

Indeed, if $\tilde{h}$ is to be finite it is clear that h(x) must <u>vanish</u> as $x \to 0$. This tells us that we must subtract out the sea and gluon contributions, consistent with the Zweig rule (28) and the notion of a bare structure function.

To proceed further, we confirm the valence normalization in (29) by first working in the weak-binding limit, with $h(x) \to \delta(x-1/2)$ predicting $\tilde{h} = 2$ and $m_\pi = 2\hat{m}_{\rm curr}$ as expected. A more sophisticated h(x), however, should recover the $(1-x)^2$ behavior[23] as $x \to 1$, leading to the "bare" structure function $\propto x^2(1-x)^2$ and the scale[8]

$$\tilde{h} = 5/2 \quad , \qquad \hat{m}_{\rm curr} = \frac{1}{\sqrt{5}}\, m_\pi \approx 62 \text{ MeV}. \tag{30}$$

Indeed there are five[8] such bare quark structure functions for 0^-, 1^-, $1/2^+$, $3/2^+$ hadron states which lead to the current quark mass scale (30) in the context of neutral PCAC.

Although this neutral PCAC scale $\hat{m}_{curr} \sim 60$ MeV is of significant interest (and will be used in the next section to demonstrate the consistency of a unified picture of current and constituent quarks), it is the proportionality of pseudoscalar masses (and not their squares) with current quark masses as in (26) that we wish to exploit here.

To this end we compute the vacuum matrix element of[24] $[Q_5^{NS}, \partial\cdot A^{NS}] = \hat{m}_{curr}(\bar{u}u + \bar{d}d)_{curr}$, for the anomalous divergence

$$\partial\cdot A^{NS} = -\hat{m}\, v^{NS} + (g^2/32\pi^2)\varepsilon_{\mu\nu\alpha\beta} G_r^{\mu\nu}\, G_r^{\alpha\beta} \ . \tag{31}$$

This leads to the U(1) vacuum Ward identity

$$\hat{m}_{curr} <0|(\bar{u}u + \bar{d}d)_{curr}|0> \ = -i\hat{m}^2_{curr} \int d^4x <0|T(v^{NS}(x), v^{NS}_{(0)})|0> + <<\nu^2>> \ , \tag{32a}$$

$$<<\nu^2>> \equiv -i(g/4\pi)^4 \int d^4x <0|T(GG^*(x), GG^*(0))|0> \ . \tag{32b}$$

Now in the neutral PCAC scheme, the LHS of (32a) is $\mathcal{O}(m^2_{curr})$ by virtue of (25). Thus it is clear that $<<\nu^2>>$ likewise satisfies

$$<<\nu^2>>_{\text{neutral PCAC}} = \mathcal{O}(\hat{m}^2_{curr}) \ . \tag{33}$$

This result is perfectly consistent with the WKB estimate of $<<\nu^2>>$, giving[4]

$$<<\nu^2>>_{WKB} = \mathcal{O}(\hat{m}^2_{curr}) \ . \tag{34}$$

Moreover the neutral PCAC result (25) is also consistent with the independent WKB estimate[4]

$$<0|(\bar{q}q)_{curr}|0>_{WKB} = \mathcal{O}(\hat{m}_{curr}) \ . \tag{35}$$

In contrast to this self-consistent neutral PCAC picture, one may consider the strong PCAC scheme which blurs the distinction between current and constituent quarks. If as in refs. 2 and 3 we take $\langle 0|(\bar{q}q)_{curr}|0\rangle \neq 0$, then the chiral breaking U(1) Ward identity (32) requires $\langle\langle \nu^2 \rangle\rangle = \mathcal{O}(\hat{m}_{curr})$. The WKB approximation contradicts this, both in (34) and in (35). Moreover none of the many contrived schemes to solve this "U(1) problem" are satisfactory.[4] With hindsight Crewther's U(1) problems are caused by his a priori insistence on the strong PCAC requirement $\langle 0|(\bar{q}q)_{curr}|0\rangle \neq 0$, which in fact conflicts with the WKB approximation. Therefore within the context of strong PCAC an alternative to the WKB approximation must be found. On the other hand as we have seen, neutral PCAC and the WKB approximation are compatible.

V. Flavor Independence of $<0|(\bar{q}q)_{con}|0>$

We have argued that $<0|(\bar{q}q)_{con}|0> \neq 0$ in the chiral limit. Since the vacuum is a unitary singletwe expect that

$$<0|(\bar{u}u)_{con}|0> \; = \; <0|(\bar{d}d)_{con}|0> = <0|(\bar{s}s)_{con}|0> \neq 0 \quad . \tag{36}$$

By way of contrast $<0|(\bar{q}q)_{curr}|0> \to 0$ in the chiral limit for neutral PCAC, but

$$<0|(\bar{u}u)/m_u)_{curr}|0> \; = <0|(\bar{d}d/m_d)_{curr}|0> = <0|(\bar{s}s)/m_s)_{curr}|0> \neq 0 \; . \tag{37}$$

To test (36) we recall from (13) that $<0|(\bar{q}q)_{con}|0>$ is proportional to the dynamically-generated quark mass $m(p^2) \to m_{dyn}$ and (12) in the chiral limit. Away from the chiral limit we might then expect that m_{dyn} measures the differences between constituent and current quark masses -- then flavor independent according to (36):

$$m_{dyn} = m_u^{con} - m_u^{curr} = m_d^{con} - m_d^{curr} = m_s^{con} - m_s^{curr} \; . \tag{38}$$

In the neutral PCAC scheme the condition (37) applied to (19) predicts the current quark mass ratio(7,8)

$$(m_s/\hat{m})_{curr} = [2(m_K^2/m_\pi^2) - 1]^{1/2} \approx 5 \; , \tag{39}$$

which in fact is consistent(25) with the πN σ-term of 65 MeV and the Goldberger-Treiman discrepancy of 6%. Combining (39) with the scale (30) gives $m_{s,curr} \simeq 300$ MeV. Subtracting these current quark masses from the known constituent quark masses of 330 MeV and 550 MeV then gives

$$m_{dyn} = \hat{m}_{con} - \hat{m}_{curr} \approx 330\text{-}60 \approx 270 \text{ MeV} \qquad (40a)$$

$$m_{dyn} = m_s^{con} - m_s^{curr} \approx 550\text{-}300 \approx 250 \text{ MeV}. \qquad (40b)$$

The important point is that (40a) is reasonably close to (40b) in magnitude. Both, in fact, are consistent with a recent calculation of m_{dyn} in a QCD context.[26]

If we extend this analysis to the charmed sector, (presumably with greater PCAC extrapolation corrections) then we find[8]

$$(m_c/\hat{m})_{curr} = [2(m_D^2/m_\pi^2) - 1]^{1/2} \approx 19, \qquad (41)$$

so that $m_c^{curr} \sim 1200$ MeV. Then for $m_c^{con} \sim 1550$ MeV, we compute

$$m_{dyn} = m_c^{con} - m_c^{curr} \sim 1550 - 1200 \sim 350 \text{ MeV}, \qquad (42)$$

also close to (40).

Lastly we may investigate the electromagnetic mass splitting of mesons and baryons to extract $(m_u - m_d)_{curr}$. In the soft meson limit the Dashen theorem[27] is exact, $(H_{JJ})_{\Delta K} = (H_{JJ})_{\Delta\pi}$, from which the $\varepsilon_3 u_3$ current quark tadpole structure of H' given by (19) leads to (in any PCAC scheme)

$$\left(\frac{m_u - m_d}{m_s + \hat{m}}\right)_{curr} \approx \frac{\Delta m_K^2 - \Delta m_\pi^2}{m_K^2} \approx 0.021 \quad . \qquad (43)$$

For the neutral PCAC mass scale of $\hat{m}_{curr} \approx 60$ MeV and (39), (43) predicts the mass shift

$$(m_u - m_d)_{curr} \approx -8 \text{ MeV}. \qquad (44)$$

In fact the observed n-p mass difference also predicts[28] (44). On the other hand, a recent constituent quark analysis of em mass splitting concludes that[29]

$$(m_u - m_d)_{con} \simeq -6 \text{ MeV}. \qquad (45)$$

Once again (44) is sufficiently close to (45) that the flavor independence of m_{dyn} (and therefore of $\langle 0|(\bar{q}q)_{con}|0\rangle$) would appear to be satisfied in nature.

Although the essence of the point we are making in this section is contained in the preceding discussion, it is worthwhile being somewhat more careful as to the meaning of the multitude of mass parameters that appear. The scale-dependence of masses (see (10) above, for example) should be examined explicitly.

The concept of a scale-dependent quark mass $m(p^2)$ originated in a field-theoretic study of inclusive lepton-hadron scattering,[30] and had the interpretation of a parton mass appropriate to the momentum transfer p^2. Georgi and Politzer[30] further proposed that this mass should interpolate between current and constituent quark masses, although the idea was not made precise. Now, in perturbation theory, it is easy to see that

$$\lim_{p^2 \to \infty} \frac{m_i(p^2)}{m_j(p^2)} = \left(\frac{m_i}{m_j}\right)_{bare} \qquad (46)$$

where i,j are flavor indices (u,d,s,...). Politzer[6] showed that this equation holds true in the presence of dynamical spontaneous symmetry breakdown, even if it is non-perturbative in origin. While we will make use of a stronger result of Politzer for $m_i(p^2)$ below, for the moment we simply draw some consequences from (46) in the context of neutral PCAC as presented in this paper. The scale-dependent quark mass $m(p^2)$ may be decomposed into a sum of two terms,

$$m_i(p^2) = m_{i,curr}(p^2) + m_{dyn}(p^2) \quad , \tag{47}$$

where $m_{curr}(p^2)$, as we have seen, arises from explicit chiral symmetry breaking and is flavor dependent while $m_{dyn}(p^2)$ is dynamically generated, nonzero even if the Lagrangian itself has no mass parameters to set a scale, and flavor independent. The p^2-independent current quark mass derived in (29) corresponds to $\tilde{h}(p^2)$ evaluated for $p^2 \sim 10\,\mathrm{GeV}^2$ region where the quark sea and gluon contributions in the pion are suppressed.

At lower p^2, however, we must invoke detailed renormalization-group formulas to recover consistency. More explicitly, we take

$$m_{i,curr}(p^2) = m_{i,curr}(M^2)\left[\frac{g^2(p)}{g^2(M)}\right]^d \tag{48a}$$

$$m_{dyn}(p^2) = \frac{4g^2(p)}{p^2} \langle 0|(\bar{q}q)_{con}(M)\,|0\rangle \quad \left[\frac{g^2(p)}{g^2(M)}\right]^{-d} \tag{48b}$$

where

$$d = \frac{12}{33 - 2N_f} , \qquad \frac{g^2}{4\pi^2} = \frac{d}{2\ln(p^2/\Lambda^2)} \tag{49}$$

where M is the renormalization mass, and $\Lambda \simeq 0.25$ GeV sets the scale of the color coupling g^2. These expressions for $m_{i,curr}$ and $m_{dyn}(p^2)$ are only correct asymptotically, so use of them for finite p^2 is questionable; nevertheless, our results will be seen to be encouraging, thus supporting the viewpoint advanced herein.

Phenomenology suggests that $\hat{m}_{con}$ is approximately 330 MeV; but what value of p^2 should one take to make use of this? Georgi and Politzer[30] proposed to define the constituent quark mass at threshold for $\bar{q}q$ production, $p^2 = 4m_i^2$. This prescription is not gauge invariant. We will choose $p^2 = m_i^2$ as in (14), the only gauge invariant definition, so

$$\hat{m}(p^2 = \hat{m}^2) = \hat{m}_{con} = 330 \text{ MeV} \quad . \tag{50}$$

As indicated by recent lattice gauge theory results[31], we will assume that $g^2(p^2)$ is essentially constant for $p^2 \leq 1 \text{ GeV}^2$. Then our estimate of 62 MeV for $\hat{m}_{curr}$ (10 GeV^2) implies $\hat{m}_{curr}$ (1 GeV^2) = 82 MeV = $\hat{m}_{curr}$ $(p^2 = \hat{m}^2_{con})$. Substituting these estimates into (48), we find

$$\frac{4g^2(M)}{M^2} \langle 0|(\bar{q}q)_{con}(M)|0\rangle = 248 \text{ MeV} \tag{51}$$

for M = 330 MeV.

Now that the scale of the flavor-independent vacuum matrix element has been set, we may use it to test our values for other current quark masses. (The calculation may equally be viewed as a check on the flavor independence of the vacuum matrix element, given our current quark masses.) For the strange quark, we demand via (47),

$$m_{s,con}(p^2 = m^2_{s,con}) = 410 \text{ MeV} + \frac{M^2}{m^2_{s,con}} \left[\frac{g^2(m^2_{s,con})}{g^2(M)}\right]^{1-d} (248 \text{ MeV}), \tag{52}$$

where we have used the assumption of constancy for g^2 at $p^2 \leq 1 \text{ GeV}^2$ and the previously obtained value for m_s (10 GeV^2) of 310 MeV, to write 410 MeV for $m_{s,curr}$ $(p^2 = m^2_{s,con})$. The solution to (52) is

$$m_{s,con} = 512 \text{ MeV} \quad , \tag{53}$$

not inconsistent with $m_{s,con}$ estimates which vary from 500 - 550 MeV. Inputing instead the strong PCAC values of $\hat{m}_{curr} \approx 5$ GeV and $m_{s,curr} \approx 150$ MeV at $p^2 \sim 10\ \text{GeV}^2$, a procedure analogous to (50)-(52) leads to the self-consistent solution $m_{s,corr} \sim 350$ MeV, significantly below the expected value.

The situation with respect to the charmed quark mass is not so straightforward. We estimated above that $m_{c,curr} = 1200$ MeV, but this was for a scale of p^2 roughly ($10\ \text{GeV}^2$). Since g^2 is expected to vary in the 1-10 GeV^2 interval, $m_{c,curr}$ will likewise vary. However, so long as the vacuum matrix element is not strongly dependent on the renormalization point, the p^2 in the denominator will make the second term in (48) negligible for the charmed quark (as well as for higher mass quarks). Therefore

$$m_{c,con}(p^2 = m^2_{c,con}) \approx m_{c,curr}(p^2 = 10\ \text{GeV}^2) \left(\frac{g^2(m^2_{c,con})}{g^2(10\ \text{GeV}^2)} \right)^d \tag{54}$$

which leads to the expected result

$$m_{c,con} \simeq 1440 \text{ MeV}$$

for a reasonable guess as to the variation of g^2.

We conclude that flavor independence of the dynamically-generated quark masses is indeed valid in the neutral PCAC scheme.

VI. Conclusion

We have observed that there is a mismatch between the simple transformation properties of hadron states under light-plane charges and the driving of PCAC under static axial charges. We maintain the consequent distinction between current and constituent quarks should not be ignored -- especially as manifested in the bad quark density operators $\bar{q}q$ and $\bar{q}\gamma_5 q$. In this paper we have seen that the corresponding vacuum matrix elements of the scalar densities have distinct attributes, according to the following scheme:

$<0|(\bar{q}q)_{con}|0>$	Static charges Nambu-Goldstone spontaneous breakdown Dynamically-generated quark mass Flavor independence
$<0|(\bar{q}q)_{curr}|0>$	Light plane charges Current algebra Chiral symmetry breaking Neutral PCAC resolution of mismatch

Such a theoretical picture is motivated by the parton model and required for a simple solution of the vacuum Ward identity U(1) problem. Moreover the neutral PCAC scheme is in phenomenological agreement with the πN σ-term, Goldberger-Treiman discrepancy, 0^-, $1/2^+$, 1^-, $3/2^+$ hadron mass spectrum and the flavor independence of the constituent to current quark mass difference. Finally, it is consistent with renormalization group mass formulae.

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